Exercise 44

Find the derivative. Simplify where possible.

$$y = \operatorname{sech}^{-1}(e^{-x})$$

Solution

Take the derivative using the chain rule.

$$\begin{aligned}
y' &= \frac{d}{dx}[\operatorname{sech}^{-1}(e^{-x})] \\
&= \left[-\frac{1}{(e^{-x})\sqrt{1-(e^{-x})^2}}\right] \cdot \frac{d}{dx}(e^{-x}) \\
&= \left(-\frac{e^x}{\sqrt{1-e^{-2x}}}\right) \cdot (e^{-x}) \cdot \frac{d}{dx}(-x) \\
&= \left(-\frac{e^x}{\sqrt{1-e^{-2x}}} \cdot \frac{e^x}{e^x}\right) \cdot (e^{-x}) \cdot (-1) \\
&= \left(-\frac{e^{2x}}{\sqrt{(1-e^{-2x})e^{2x}}}\right)(-e^{-x}) \\
&= \left(-\frac{e^{2x}}{\sqrt{e^{2x}-1}}\right)(-e^{-x}) \\
&= \frac{e^x}{\sqrt{e^{2x}-1}}
\end{aligned}$$